Determine if each of the following functions is continuous. **STATE YOUR CONCLUSIONS CLEARLY.** SCORE: _____ / 6 PTS
If a function is continuous, justify your conclusion using the definition(s) and/or theorems.
If a function is not continuous, show clearly which part of the definition of "continuous" is not true.

[a] $f(x) = \begin{cases} x^3 - 2x^2 - 8, & \text{if } x \le 3 \\ x^3 - 4x^2 + 10, & \text{if } x > 3 \end{cases}$

[b] $f(x) = \begin{cases} \frac{x^3+1}{x^2-1}, & \text{if } x < -1 \\ \frac{x^2-4}{x+3}, & \text{if } x > -1 \end{cases}$

$\boxed{f(-1) \text{ DNE} \quad f \text{ NOT CONT.}}$ 1

$\boxed{f \text{ IS CONT. AT } x \ne 3 \text{ (POLYNOMIAL)}}$ 1

$f(3) = 3^3 - 2(3)^2 - 8 = 1$

$\frac{1}{2}$ $\boxed{\lim_{x \to 3^+} (x^3 - 4x^2 + 10) = 3^3 - 4(3)^2 + 10 = 1}$

$\frac{1}{2}$ $\boxed{\lim_{x \to 3^-} (x^3 - 2x^2 - 8) = 3^3 - 2(3)^2 - 8 = 1}$

1 $\boxed{\lim_{x \to 3} f(x) = 1} = \boxed{f(3)}$ 1 $\rightarrow f$ IS CONT. AT $x = 3 \rightarrow \boxed{f \text{ IS CONT.}}$ 1

Let $f(x) = \sqrt{19 - 3x}$. SCORE: _____ / 8 PTS

[a] Find $f'(x)$.

1 $\boxed{\lim_{h \to 0} \frac{\sqrt{19-3(x+h)} - \sqrt{19-3x}}{h}} \cdot \frac{\sqrt{19-3(x+h)} + \sqrt{19-3x}}{\sqrt{19-3(x+h)} + \sqrt{19-3x}}$

$= \lim_{h \to 0} \frac{19 - 3(x+h) - (19 - 3x)}{h(\sqrt{19-3(x+h)} + \sqrt{19-3x})}$

2 $= \boxed{\lim_{h \to 0} \frac{-3h}{h(\sqrt{19-3(x+h)} + \sqrt{19-3x})}} = \boxed{\frac{-3}{2\sqrt{19-3x}}}$ 1

[b] Find the slope-point form of the equation of the tangent line to the curve of $f(x)$ at the point where $x = 1$.

$f'(1) = \frac{-3}{2\sqrt{16}} = \boxed{-\frac{3}{8}}$ 1 $\boxed{y - 4 = -\frac{3}{8}(x - 1)}$ 1

[c] The position (in yards) of an object moving in a straight line is given by $s(t) = \sqrt{19 - 3t}$, where t is the time in minutes.
Find the instantaneous velocity of the object at time $t = 5$. Give the correct units for your answer.

$s'(5) = \frac{-3}{2\sqrt{4}} =$ 1 $\boxed{-\frac{3}{4}}$ $\boxed{\text{YARD/MINUTE}}$ 1

Using complete sentences and proper mathematical notation, write the formal definition of "derivative (function)". SCORE: _____ / 1 PT

THE DERIVATIVE OF f IS $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

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Using complete sentences and proper mathematical notation, write the formal definition of "continuous (at a point)". **SCORE: _____ / 2 PTS**

f IS CONTINUOUS AT a IFF $f(a)$ EXISTS,

$\lim\limits_{x\to a} f(x)$ EXISTS AND

$\lim\limits_{x\to a} f(x) = f(a)$

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The amount of sugar that can be dissolved in a cup of coffee depends on the temperature of the coffee. **SCORE: _____ / 2 PTS**
Suppose $s = f(t)$, where s is the amount of sugar that can be dissolved (in grams), and t is the termperature (in degrees Celsius).

[a] What does $f(80) = 60$ mean ? Give the correct units for all numbers in your answer.

60g OF SUGAR CAN BE DISSOLVED IN A CUP OF 80°C COFFEE

[b] What does $f'(80) = 2$ mean ? Give the correct units for all numbers in your answer.

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IF A CUP OF COFFEE IS 80°C,
≈2 ADDITIONAL GRAMS OF SUGAR CAN BE DISSOLVED IN IT
FOR EACH 1°C HOTTER THE COFFEE GETS

Find the following limits. **SCORE: _____ / 7 PTS**
Each answer should be a number, ∞, $-\infty$, or DNE (only if the other answers do not apply).

[a] $\lim\limits_{x\to\infty} \tan^{-1} x$

$= \boxed{\frac{\pi}{2}}$ 1

[b] $\lim\limits_{x\to-\infty} \frac{\sqrt{25x^2-16x}}{7-3x} \cdot \frac{\frac{1}{x}}{\frac{1}{x}}$

$= \lim\limits_{x\to-\infty} \frac{\sqrt{25x^2-16x}\cdot -\sqrt{\frac{1}{x^2}}}{\frac{7}{x}-3}$

1 $= \lim\limits_{x\to-\infty} \boxed{\frac{-\sqrt{25-\frac{16}{x}}}{\frac{7}{x}-3}}$ 2

$= \frac{-\sqrt{25-0}}{0-3} = \boxed{\frac{5}{3}}$ 1

[c] $\lim\limits_{x\to\infty} \arccos e^{-x}$

$= \boxed{\arccos 0}$ 1

$= \boxed{\frac{\pi}{2}}$ 1

Prove that $\ln x = \frac{1}{x}$ for some x in the interval $(1, e)$. **DO NOT ATTEMPT TO SOLVE FOR *x*.** **SCORE: _____ / 4 PTS**

1 LET $f(x) = \ln x - \frac{1}{x}$. f IS CONTINUOUS ON $[1, e]$ SINCE IT IS THE DIFFERENCE OF CONTINUOUS FUNCTIONS.

$f(1) = \ln 1 - \frac{1}{1} = 0 - 1 = -1$ AND $f(e) = \ln e - \frac{1}{e} = 1 - \frac{1}{e} = \frac{e-1}{e}$

$\boxed{-1 < 0 < \frac{e-1}{e}}$, SO $\boxed{\text{BY IVT}}$ $\boxed{\text{FOR SOME } c \in (1,e),\ f(c) = 0 \text{ IE. } \ln c - \frac{1}{c} = 0 \text{ OR } \ln c = \frac{1}{c}}$ 1

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